

# Dynamical Origin of Seesaw

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In anomaly free gauged three-flavor  $SU(3)_f \times SU(2)_L \times U(1)_Y$  model of Yanagida with fermion and gauge boson masses described by conveniently chosen elementary scalar Higgs fields the neutrino mass matrix comes out in the seesaw form. Following Yanagida's suggestion we demonstrate that no Higgs fields are needed. *Strong flavor gluon interactions themselves*, treated in a separable approximation, result in universally split lepton and quark masses calculated in terms of a few parameters. While the realistic splitting of charged lepton and quark masses requires the electroweak and QCD radiative corrections the neutrino seesaw mass matrix comes out exact.

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## I. INTRODUCTION

Understanding the measured wide and irregular charged lepton and quark mass spectrum is a nightmare of theoretical elementary particle physics. Understanding the extreme lightness of the observed neutrinos is its almost unbearable stage. Description of these phenomena, however, does exist. The charged lepton and quark masses are described in the Standard model by the Higgs mechanism [1], and the extremely light neutrinos are described in its minimal extension by the seesaw [2]. It amounts to postulating superheavy Majorana masses  $M_M$  of three right-handed sterile neutrinos  $\nu_R$ ; the observed neutrino spectrum is given by the diagonalized  $6 \times 6$  symmetric matrix

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_M \end{pmatrix} \quad (1)$$

where  $m_D$  are three Dirac neutrino masses generated by the ordinary Higgs mechanism. As a result the physical neutrino spectrum consists of three superheavy Majorana neutrinos with masses  $\sim M_M$ , and three active Majorana neutrinos with masses  $m_\nu \sim m_D^2/M_M$ . Choosing the free parameters  $m_D$  and  $M_M$  appropriately we obtain the masses of three active (Majorana) neutrinos at will.

This elegant phenomenological construction rises questions. First, are there deeper reasons for postulating superheavy sterile neutrinos? If not, the very explanation of extreme lightness of the observed neutrinos by postulating the existence of unobservably heavy ones does not seem very deep. Second, why zero in the left upper corner instead of the Majorana mass matrix  $m_M$  of the left-handed neutrinos of the Standard model?

The existence of superheavy right-handed sterile neutrinos is perfectly justified in the gauged three-flavor  $SU(3)_f \times SU(2)_L \times U(1)_Y$  model of T. Yanagida [3]. First, the very existence of right-handed neutrinos is enforced by the strong theoretical requirement of anomaly freedom. Second, their massiveness is enforced by the experimental fact that the gauge flavor  $SU(3)_f$  symmetry is badly broken and yet unobserved. The spontaneous

breakdown of the gauge chiral  $SU(3)_f \times SU(2)_L \times U(1)_Y$  symmetry down to  $U(1)_{em}$  is phenomenologically described by the following weakly interacting condensing Higgs fields: (1) The field  $\chi^{ab}(6, 1, 0)$  gives rise to huge different masses of all eight flavor gluons. Its Yukawa coupling with sterile right-handed neutrinos and their charge conjugates generates their huge Majorana masses. (2) The fields  $\phi^a(8, 2, 1)$  and  $\phi^0(1, 2, 1)$  are responsible mainly for the electroweak symmetry breakdown. The values in parentheses are the representation dimensions of  $SU(3)_f$ ,  $SU(2)_L$  and the values of the weak hypercharge, respectively. The Majorana mass matrix  $m_M$  of the left-handed neutrinos stays zero because the elementary Higgs field  $\phi(6, 3, -2)$  which would generate it [4] was deliberately not introduced.

In conclusion of [3] Yanagida notices that his model "is a possible candidate for the spontaneous mass generation by dynamical symmetry breaking [5]". Indeed, without scalar sector the gauge  $SU(3)_f \times SU(2)_L \times U(1)_Y$  model would yield the fermion mass spectrum calculable in terms of a few free parameters: (1) The coupling constant  $h$  of the strongly coupled  $SU(3)_f$  or, due to the dimensional transmutation, its scale  $\Lambda$ . (2) The coupling constants  $g$  and  $g'$  of the weakly coupled  $SU(2)_L$  and  $U(1)_Y$ , respectively. The numerical values of weak hypercharges  $Y_f$ ,  $f = l_L, e_R, \nu_R, q_L, u_R, d_R$  of the chiral electroweakly interacting fermion fields  $f$  are not free parameters. They are uniquely fixed by their electric charges  $Q = T_3 + \frac{1}{2}Y$ :

$$\begin{aligned} Y(l_L) &= -1, & Y(e_R) &= -2, & Y(\nu_R) &= 0 \\ Y(q_L) &= \frac{1}{3}, & Y(u_R) &= \frac{4}{3}, & Y(d_R) &= -\frac{2}{3}. \end{aligned}$$

We have demonstrated in detail elsewhere [6] that the strong exchanges of the dynamically massive flavor gluons indeed play the role of the extended Higgs sector of the Yanagida model. Here we emphasize their selective dynamical role which results in the computable neutrino mass spectrum in the seesaw form.

## II. FERMION MASS SPECTRUM

Our task is to generate the fermion proper self-energy  $\Sigma(p)$  in the full fermion propagator

$$S^{-1}(p) = \not{p} - \Sigma(p) \quad (2)$$

$$\Sigma(p) = 3 \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{h}_{ab}^2 ((p-k)^2)}{(p-k)^2} T_a(R) \Sigma(k) [k^2 - \Sigma^+(k) \Sigma(k)]^{-1} T_b(L) \quad (3)$$

depicted in Fig.1.

To make the formidable task of the dynamical fermion mass generation tractable we resort to approximations. First, without loss of generality we fix the external (Euclidean) momentum as  $p = (p, \vec{0})$ , and integrate over angles. Integration over the momenta only up to  $\Lambda$  means that the resulting model is not asymptotically, but strictly free above this scale:

$$\Sigma(p) = \int_0^\Lambda k^3 dk K_{ab}(p, k) T_a(R) \Sigma(k) [k^2 + \Sigma^+ \Sigma]^{-1} T_b(L). \quad (4)$$

Because of the unknown behavior of  $\bar{h}_{ab}^2$  below  $\Lambda$  the kernel  $K_{ab}(p, k)$ , separately symmetric in momenta and flavor octet indices, is *entirely unknown*. Second and most

dynamically by the strong flavor gluon interactions of  $SU(3)_f$ . In general  $\Sigma(p)$  is a complex  $3 \times 3$  matrix, the solution of the Schwinger-Dyson equation [7]

important, for this unknown kernel we make a separable Ansatz

$$K_{ab}(p, k) = \frac{3}{4\pi^2} \frac{g_{ab}}{pk}. \quad (5)$$

It is conceptually quite similar to the separable Ansatz for the potential in the BCS superconductor which leads to the explicit solution of the BCS gap equation [8]. Why the simple separable approximation taking into account only the opposite momenta around Fermi surface is phenomenologically so successful was a mystery for years. It was theoretically clarified much later by Polchinski [9].

Here  $g_{ab}$  are the effective dimensionless low-energy parameters which characterize the *complete spontaneous breakdown of the gauge*  $SU(3)_f$ . Very important question is how many independent parameters are necessary for such a breakdown. We believe the answer is *three*. We present two independent explicit illustrations of this statement.

First illustration is provided by the Higgs mechanism for  $SU(3)_f$  with the condensing scalar sextet  $\chi$ , the complex symmetric  $3 \times 3$  matrix [3, 6]. Here the spontaneous gauge boson and fermion mass generations are two generically independent phenomena: (i) Eight different gauge boson masses squared are expressed in terms of three parameters  $(v_1, v_2, v_3)$ , the diagonal vacuum expectation values of  $\chi$ :

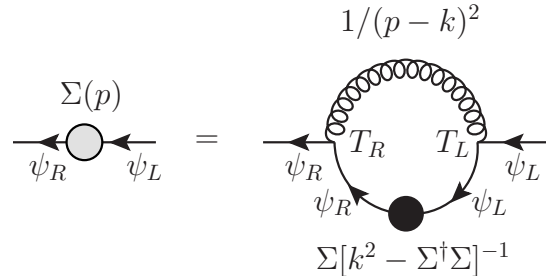


FIG. 1: At high momenta the coupling matrix  $\bar{h}_{ab}^2$  is small and known (asymptotic freedom). Below  $\Lambda$  the coupling matrix  $\bar{h}_{ab}^2$  is large and entirely unknown.

$$M^2 \sim \begin{bmatrix} (v_1 + v_2)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (v_1 - v_2)^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2(v_1^2 + v_2^2) & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}}(v_1^2 - v_2^2) \\ 0 & 0 & 0 & (v_1 + v_3)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (v_1 - v_3)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (v_2 + v_3)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (v_2 - v_3)^2 & 0 \\ 0 & 0 & \frac{2}{\sqrt{3}}(v_1^2 - v_2^2) & 0 & 0 & 0 & 0 & \frac{2}{3}(v_1^2 + v_2^2 + 4v_3^2) \end{bmatrix}$$

(with diagonalization of the  $(3, 8)$  block understood). (ii) The  $SU(3)_f$  invariant Yukawa interaction with the sextet  $\chi$  gives rise to three different Majorana masses to chiral neutrinos proportional to  $v_i$  provided they transform as a triplet of  $SU(3)_f$ .

Second illustration is provided by spontaneous breakdown of the gauge  $SU(3)_f$  by the dynamically generated Majorana masses of neutrinos transforming as the triplet [6]. Here the phenomena of spontaneous generation of the gauge and the fermion masses are self-consistently related. Three different Majorana masses are found by solving the SD equation, and eight gauge boson masses are the functions of these three Majorana masses computed by the Pagels-Stokar formula.

We conclude that the matrix  $g_{ab}$  in the present 'semi-microscopic' approach should be expressible as a function of three dimensionless parameters. The matrix  $g \sim M^2/\Lambda^2$  is the simplest weak-coupling prototype of such a relation, with positive-definiteness relaxed. Ultimately, however, because the group  $SU(3)$  is simple, even these three parameters should be calculable.

In separable approximation the SD equation is immediately formally solved:

$$\Sigma(p) = \frac{\Lambda^2}{p} T_a(R) \Gamma_{ab} T_b(L) \equiv \frac{\Lambda^2}{p} \sigma \quad (6)$$

The difficult part is that the numerical matrix  $\Gamma$  has to fulfil the homogeneous nonlinear algebraic self-consistency condition (gap equation).

The obtained momentum dependence of  $\Sigma(p) \sim 1/p$  is not without support. Because the masses of flavor gluons come out huge it is justified to think heuristically of the dynamically generated fermion masses in terms of the four-fermion interaction of Nambu and Jona-Lasinio [5]. In a series of papers [10] Philip Mannheim argues that the theoretically consistent treatment of the fermion mass generation by the four-fermion dynamics should result in  $\Sigma(p) \sim 1/p$ .

### 1. Fermion masses with flavor mixing

**For the right-handed neutrinos** the effective Majorana mass term has the form

$$\mathcal{L}_{Majorana} = -\frac{1}{2} (\bar{\nu}_R \Sigma_M(p) (\nu_R)^C + h.c.) \quad (7)$$

In flavor space it therefore transforms as

$$\bar{3} \times \bar{3} = 3_a + \bar{6}_s \quad (8)$$

where the subscripts abbreviate the antisymmetric (a) and symmetric (s) representations.

Because of the Pauli principle  $\sigma_M$  in (6) is a general complex symmetric  $3 \times 3$  matrix of the sextet. It can be

put into a positive-definite real diagonal matrix  $\gamma^M$  by a constant transformation

$$\sigma_M = U^+ \gamma^M U^*. \quad (9)$$

Then, the gap equation becomes

$$\gamma^M = U T_a(R) U^+ g_{ab} I(\gamma^M) U^* T_b(L) U^T \quad (10)$$

where

$$I(\gamma) = \frac{3}{16\pi^2} \gamma \int_0^1 \frac{dx}{x + \gamma^2} = \frac{3}{16\pi^2} \gamma \ln \frac{1 + \gamma^2}{\gamma^2}. \quad (11)$$

The diagonal entries of the equation (10) determine the sterile neutrino masses, the nondiagonal entries provide relations for the mixing angles and the new CP-violating phases. These phases are most welcome as a source of an extra CP violation needed for understanding of the baryon asymmetry of the Universe [11]. Because  $\nu_R$  transforms as a triplet,  $T_a(R) = \frac{1}{2} \lambda_a$ . The charge-conjugate  $\nu_R$  is a left-handed field i.e.,  $(\nu_R)^C = (\nu)_L^C$ . Because charge conjugation is essentially the hermitian conjugation,  $T_b(L) = -\frac{1}{2} \lambda_b^* = -\frac{1}{2} \lambda_b^T$  are the generators of antitriplet.

**For Dirac fermions** the effective Dirac term has the form

$$\mathcal{L}_{Dirac} = -(\bar{f}_R \Sigma_D(p) f_L + h.c.) \quad (12)$$

In flavor space it therefore transforms as

$$\bar{3} \times 3 = 1 + 8. \quad (13)$$

The general complex  $3 \times 3$  matrix  $\sigma_D$  can be put into a positive-definite real diagonal matrix  $\gamma^D$  by a constant bi-unitary transformation:

$$\sigma_D = U^+ \gamma^D V. \quad (14)$$

The gap equation becomes

$$\gamma^D = U T_a(R) U^+ g_{ab} I(\gamma^D) V T_b(L) V^+. \quad (15)$$

The diagonal entries of the equation (15) determine the fermion masses, the nondiagonal entries provide relations for the CKM mixing angles and the SM CP-violating phase. Because all chiral fermion fields of the model transform as triplets,  $T_a(R) = \frac{1}{2} \lambda_a$  and  $T_b(L) = \frac{1}{2} \lambda_b$ .

### 2. Majorana masses $M_M$ and Dirac masses $m_D$

In the following we set the fermion mixing matrices to the unit matrix and show that the Majorana masses  $M_M$  of the right-handed neutrinos come out naturally huge of order  $\Lambda$ , whereas the Dirac masses of the fermions of the

model are naturally small compared to  $\Lambda$ . 'Naturally' means that in the Lagrangian there are no parameters (effective couplings  $g_{ab}$ ) which would differ by many orders of magnitude. Huge mass ratios emerge only *in solutions* of the underlying field equations.

Without mixing the Majorana and Dirac gap equations are [6]

$$\gamma^M = -\frac{1}{4}\lambda_a g_{ab} I(\gamma^M) \lambda_b^T \quad (16)$$

and

$$\gamma^D = \frac{1}{4}\lambda_a g_{ab} I(\gamma^D) \lambda_b, \quad (17)$$

respectively. Because  $\Sigma(p) \equiv \frac{\Lambda^2}{p}\gamma$ , the fermion mass,

both Dirac and Majorana, defined as a pole of the full fermion propagator is

$$m \equiv \Sigma(p^2 = m^2) = \Lambda\gamma^{1/2}$$

With  $g_{11}, g_{22}, g_{33}, g_{38}, g_{44}, g_{55}, g_{66}, g_{77}, g_{88}$  different from zero the right hand sides of equations (17) and (16) are the diagonal matrices. The equations themselves can be rewritten as

$$\gamma_i^{D/M} = \sum_{k=1}^3 \alpha_{ik}^{D/M} \gamma_k^{D/M} \ln \frac{1 + (\gamma_k^{D/M})^2}{(\gamma_k^{D/M})^2} \quad (18)$$

where

$$\alpha^{D/M} = \frac{3}{64\pi^2} \begin{pmatrix} \pm \left( g_{33} + \frac{2}{\sqrt{3}}g_{38} + \frac{1}{3}g_{88} \right) & g_{22} \pm g_{11} & g_{55} \pm g_{44} \\ g_{22} \pm g_{11} & \pm \left( g_{33} - \frac{2}{\sqrt{3}}g_{38} + \frac{1}{3}g_{88} \right) & g_{77} \pm g_{66} \\ g_{55} \pm g_{44} & g_{77} \pm g_{66} & \pm \frac{4}{3}g_{88} \end{pmatrix} \quad (19)$$

and the upper and lower signs correspond to the Dirac fermion masses and the Majorana neutrino masses, respectively.

The form of the matrix  $\alpha$  suggests simplifications. In the following we take only

$$g_{33}, g_{38}, g_{88}; g_{11} = -g_{22}, g_{44} = -g_{55}, g_{66} = -g_{77}$$

different from zero (and expect that they are not independent).

(A) The matrix gap equation for the Dirac masses  $m_{iD}$  becomes diagonal and decoupled, and it is easily solved. Provided the combinations

$$\begin{aligned} \alpha_1 &= \frac{3}{64\pi^2} (g_{33} + \frac{2}{\sqrt{3}}g_{38} + \frac{1}{3}g_{88}) \\ \alpha_2 &= \frac{3}{64\pi^2} (g_{33} - \frac{2}{\sqrt{3}}g_{38} + \frac{1}{3}g_{88}) \\ \alpha_3 &= \frac{3}{64\pi^2} \frac{4}{3}g_{88} \end{aligned}$$

are all positive and all  $\alpha_i \ll 1$ , the resulting universal

flavor-splitting Dirac mass formula is

$$\boxed{m_{iD} = \Lambda \exp(-1/4\alpha_i)} \quad (20)$$

(B) Finding the solution for the Majorana masses is less straightforward and we have to resort to simple numerical demonstration. First, for  $g_{11} = g_{44} = g_{66} = 0$ , the gap equations for the Majorana masses have no solution because of the minus sign in front of the  $\alpha_i$ . Consequently,  $(g_{11}, g_{44}, g_{66}) \neq 0$ . Second, in the case of sterile Majorana neutrinos we are not aware of the necessity of the hierarchical mass spectrum. With the constants  $\alpha_i$  fixed by the numerical values of the Dirac masses the equations (18) for  $\gamma_i^M$  can be viewed as a system of three inhomogeneous linear equations for the unknown  $(g_{11}, g_{44}, g_{66})$ :

$$-\frac{1}{2} \begin{pmatrix} I(\gamma_2^M) & I(\gamma_3^M) & 0 \\ I(\gamma_1^M) & 0 & I(\gamma_3^M) \\ 0 & I(\gamma_1^M) & I(\gamma_2^M) \end{pmatrix} \begin{pmatrix} g_{11} \\ g_{44} \\ g_{66} \end{pmatrix} = \begin{pmatrix} \gamma_1^M + \frac{16\pi^2}{3}\alpha_{11}^D I(\gamma_1^M) \\ \gamma_2^M + \frac{16\pi^2}{3}\alpha_{22}^D I(\gamma_2^M) \\ \gamma_3^M + \frac{16\pi^2}{3}\alpha_{33}^D I(\gamma_3^M) \end{pmatrix}.$$

This set of equations has a solution for any set of  $\gamma_i^M > 0$ .

For illustration that

$$\boxed{M_{iM} \sim \Lambda} \quad (21)$$

we put  $(\gamma_1^M, \gamma_2^M, \gamma_3^M) = (0.1, 0.2, 0.3)$  and  $(\gamma_1^D, \gamma_2^D, \gamma_3^D) = (10^{-20}, 10^{-22}, 10^{-26})$ . This corre-

sponds approximately to the hierarchy for charged leptons provided  $\Lambda = 10^{10}$  GeV. Then

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$$g = \begin{pmatrix} 8.08101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -8.08101 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.7425 & 0 & 0 & 0 & 0 & 0.0899893 \\ 0 & 0 & 0 & -21.8124 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 21.8124 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -34.029 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 34.029 & 0 \\ 0 & 0 & 0.0899893 & 0 & 0 & 0 & 0 & 1.31887 \end{pmatrix}$$


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It is important that the precise size and hierarchy of  $\gamma_i^D$  does not play any important role for the numerical values of  $\gamma_i^M$ .

It is easy to understand qualitatively the enormous, yet natural difference between the huge Majorana masses  $M_R$  of the right-handed neutrinos and the tiny Dirac masses  $m_D$  of leptons and quarks: The difference between  $\bar{3} \times \bar{3}$  and  $\bar{3} \times 3$  translates into different combinations of parameters in  $g_{ab}$  which determine  $M_R$  and  $m_D$ , respectively.

### 3. Why not $m_M$ ?

The dynamically generated fermion masses in the chiral  $SU(3)_f \times SU(2)_L \times U(1)_Y$  gauge model imply spontaneous breakdown of this symmetry down to the unbroken  $U(1)_{em}$ . Consequently, there must be exactly  $8 + 3$  'would-be' NG bosons. With the weakly coupled Higgs sector in the Yanagida model they are pre-prepared in the elementary scalar Higgs fields  $\chi^{ab}(6, 1, 0)$ ,  $\phi^a(8, 2, 1)$  and  $\phi^0(1, 2, 1)$ . We have confirmed in [6] by analyzing the *symmetry structure* of the NG poles in Ward-Takahashi identities that these 'would-be' NG bosons are in fact parts of the fermion-antifermion composites

$$\begin{aligned} \chi^{ab}(6, 1, 0) &\sim \nu_R^a(\bar{\nu}_R)^{cb} \\ \phi^a(8, 2, 1) &\sim (\bar{d}_R \lambda^a q_L + \bar{e}_R \lambda^a l_L) \\ \phi^0(1, 2, 1) &\sim (\bar{d}_R q_L + \bar{e}_R l_L) \end{aligned}$$

formed by strong exchanges of dynamically massive flavor gluons. The remaining components show up in the spectrum as massive composite Higgs-like particles.

Why not the elementary Higgs field  $\phi(6, 3, -2)$  [4]? Simply because such a decision is solely in hands of the model builder.

In model with the Higgs sector replaced by strong exchanges of dynamically massive flavor gluons the situation is entirely different. If allowed these exchanges would

obviously generate  $m_M = M_M$  which would entirely ruin the seesaw construction. We are obliged to *assume* the dynamical generation of  $m_M$  and check, if possible, whether such an assumption is theoretically consistent. Symmetry considerations themselves cannot replace the issues of dynamics.

(i) The assumption implies specific pattern of spontaneous breakdown of  $SU(3)_f \times SU(2)_L \times U(1)_Y$  symmetry.

(ii) Pauli principle and the symmetry structure of the NG poles in the WT identity suggest that the NG bosons are the composites of the lepton doublet and its charge conjugate

$$\phi^{ab}(6, 3, -2) \sim (\bar{l}_L)^{ca} i\tau_2 \bar{l}_L^b \quad (22)$$

carrying the electric charges  $(0, +, ++)$ .

(iii) Strong Coulomb repulsion in the doubly charged component *without confining force* suggests that the composite  $\phi(6, 3, -2)$  should not be formed. Consequently, neither the condensate of its neutral component,  $m_M$ , should be dynamically generated.

(iv) It is mandatory to check whether the dynamical argument applies also to other composite NG bosons. In  $\chi$  all constituents are electrically neutral and there is no problem. In singly charged components of both  $\phi^a$  and  $\phi$  made of a neutrino and a charged lepton there is no problem. In their singly charged quark components there is a strong Coulomb repulsion in  $(\bar{d}u)$ . It is, however, safely overwhelmed by the QCD confining force as in ordinary electrically charged hadrons.

### III. CONCLUSION

Not surprisingly, the obtained flavor splitting of the fermion masses (20) is universal for all types of fermions. With electroweak gauge interactions not actively involved there is nothing in the model which would distinguish between fermions with different weak hypercharges or electric charges i.e., in given generation all

Dirac masses must come out equal. This is, nevertheless, enough to break down spontaneously the gauge symmetry  $SU(3)_f \times SU(2)_L \times U(1)_Y$  down to  $U(1)_{em}$ . The electroweak gauge interactions, treated as weak external perturbations, thus become short-range:  $W$  and  $Z$  absorb the underlying composite multi-component 'would-be' NG bosons as their longitudinal polarization states and the Weinberg relation  $m_W/m_Z = \cos \theta_W$  is exact.

Since the electroweak gauge interactions remain weak up to the Planck scale they themselves cannot generate the fermion masses dynamically as suggested long ago [12]. They do, however, provide *electroweak radiative corrections* to the kernel of the SD equation which distinguish the masses  $m_{il}$  of charged leptons and masses  $m_{iu}$  and  $m_{id}$  of quarks with charges  $2/3$  and  $-1/3$ , respectively. At short distances there is also the universal positive QCD radiative correction which makes quarks in given generation heavier than the corresponding leptons. It is important that these corrections do not contain any new free parameters [13]. We believe that they should be greatly amplified by the exponentials in the universal formula (20). How to incorporate them into the separable Ansatz in the SD equation remains to be demonstrated.

We do know, however, that the radiative corrections mentioned above vanish for the Dirac neutrino masses:  $Y(\nu_R) = 0$  and there are no QCD corrections. Therefore, within the given rules of the game the computation of  $m_{iD}$  is the exact computation of the *Dirac neutrino mass matrix*. Because of sterility there are no electroweak corrections to  $M_M$  either. Hence the computation of the whole neutrino seesaw mass matrix by strong flavor gluon exchanges treated in a separable approximation is 'exact'.

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